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TYPICAL AND ATYPICAL DEVELOPMENT OF BASIC NUMERICAL ABILITIES – THE DIAGNOSTICS OF DYSCALCULIA

Summary

of the Ph.D. Thesis

2013.

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1. INTRODUCTION – ADDRESSING THE PROBLEM

In Hungary in the academic year 2011-12 (KSH, 2012), 748 thousand children attended elementary school education, of whom 52 thousand (6,95%) show ***special education needs*** (SEN). According to the 2011 Public Education Law, children with severe learning disorders, for instance the serious learning disorder in **mathematics (*developmental dyscalculia*, DC)**, belong into this category. It is the right of the rehabilitation expert committee examining learning abilities to declare that, and, in addition to pass a judgement on the condition of the child, to determine the necessary conditions and professional services needed for the education and training of the SEN child (Law CXC. 4.§/23). However, currently there are no suitable, standard procedures available for the examination of atypical development. The mostly applied, special education-oriented diagnostic method in Hungary, the ***Pedagogical Assessment of Dyscalculia*** (Dékány&Juhász, 2007) is about to undergo renewal, determining and standardizing its objective evaluation criteria (Csonkáné Polgárdi, 2012).

Learning difficulties in mathematics, leading to low performance in mathematics may be caused by a number of reasons. In order to design an effective intervention it is especially important to differentiate reliably between dyscalculia originating from the atypical development of the nervous system and lower performance due to the lower intelligence (IQ between 70 and 80), cultural handicap, shortfalls in educations, motivational-emotional problems and a slower development. A test measuring numerical abilities would be needed, which is less sensitive towards the individual differences resulting from the general cognitive abilities, the educational experiences, differences in the frequency of practice, nevertheless reliably indicates the deficit in acquiring arithmetical skills based on the internal dysfunction of basic numerical cognition (difficulty understanding simple number concepts, lack an intuitive grasp of numbers).

2. PRESENTING THE *MINIMATH* TEST

The focus of the dissertation lies in presenting the computerized test *MiniMath*, on which my colleagues and I have worked for almost ten years. To describe the contents of the test and its elaboration is not possible within the frameworks of the theses due to the lack of space; therefore I only give an account of the basic principles of the test and an overview of the numerical tasks.

At the beginning of our work we had to answer the following questions, which have guided us over the process of designing and developing to this day:

1. What shall we measure?

- **Scientific basis**: the starting point of the test was the *triple code model* of Dehaene (1992, 2003) but in defining the measured abilities and in shaping the various test situations we have also utilized the results of developmental cognitive neuroscience as well.
- **Basic counting abilities**: the numerical tasks mainly refer to the domain-specific abilities, e.g. basic number processing, which gives a differentiated picture about the number module.
- **Non—numerical abilities**: the test also measures spatial-visual abilities, which, according to the *triple code model* of Dehaene (1992, 2003) could be affected in children with DC.

2. What is the goal of measurement?

- **Screening:** the test differentiates in the lower and middle range of counting abilities.
- **Early identification**: a version of the test can be applied already prior to the start of formal mathematical instruction (from the age of five), so the atypical development of counting can be identified at an early stage.
- **Identification of sub-types**: the complex test battery enables us to differentiate between the various sub-types of background deficit, which may also help in defining the direction of therapy.

3. How should we measure?

- **Computerized test:** this way testing will be objective and precise (the latency of response can be recorded by milliseconds) and not in the least, it is appealing to children
- **Measuring reaction time**: reaction time in basic numerical tasks provides most of the information about the functioning of the number system (besides accuracy of the response).
- **Methodological approach**: in the process of working out the test battery we have tried our best to avoid methodological mistakes, bias, and maximized the reliability of the measurement.

4. What should the test look like?

- **Playfulness**: we were aiming to recall the atmosphere of a computer game. The tasks are presented embedded in a framework story, so their solution makes sense of some sort. The graphic look of the test is also playful, while not compromising the measurement in any way (e.g. the pictures should not divert the attention of the children).

– **Adaptivity**: in order to reduce the frustration caused by the lack of success the tasks and examples presented match the age of the children tested (therefore there are two age group versions of the test designed), their instruction level (e.g. Arabic number knowledge, arithmetic operations they have studied) and their performance. By inserting pre-conditions the completion time of the test is also being shortened.

Based on these principles, we have designed *MiniMath battery,* which is the collection of *MiniMath* tasks in very detail. We can measure the mathematical performance according to the age of the test person (test battery for age 5-7 and for 8-11). They differ in the scope of measured number skills, and the range of numbers involved in the tasks. We have grouped the tasks in four categories:

1. ***Quantitative judgement*** tasks include non-symbolic stimuli, e.g. they require enumerating sets including different items, and completing various operations with them: counting, error-detection in counting, comparing quantities of sets, number conservation, and informal understanding of fractions/division.

2. ***Arithmetic facts – number knowledge*** tasks require the knowledge and retrieval of information relating to numbers: transcoding, understanding quantity-words, parity judgement, retrieval of addition and multiplication facts, verification of additions and multiplications, and facts of everyday life relating to numbers.

3. ***Number concept - the meaning of numbers*** tasks require the knowledge of ordinality related to number pairs (comparing numbers) and to number sequences: recognition and formation of number sequences, Numerical Stroop, estimation of number-to-position and position-to-number on the 0-1000 number line.

4. ***Arithmetic algorithms – basic operations*** tasks require the conceptual understanding of the four basic arithmetic operations and the knowledge of their procedure: execution of addition, subtraction, multiplication up to 1000, algorithm inversion (e.g. A+B-B vs. A+A-B), knowledge of the operation signs, and semantic elaboration of the operations.

In the methodological chapter of my dissertation I take a closer look at the behavioural measurement of basic numerical skills (test situations and indicators), which we have also applied in our own test. This is followed by a detailed overview of frequently applied, scientifically based, international DC tests (*AIMSweb®TEN, TEDI-MATH, Utrecht-ENT, NUCALC, DC-Screener*), which could in the future be used to validate *MiniMath* with (Clarke & Shinn, 2002, Desoete, 2007, Van de Rijt, Van Luit & Pennings, 1999, Koumoula et.al., 2004, Butterworth, 2003. In the comparative analyses of them I have also included the Hungarian DC examination procedures (*Pedagogical Assessment of Dyscalculia* by Dékány & Juhász (2007) and the *Test Battery of Arithmetic Skills* used by Attila Márkus, 2007), so the basic principles of *MiniMath* are placed in context and warranted.

3. The Theoretical Framework of the Research

In the theoretical chapters of the dissertation I review the recent cognitive neuropsychological literature on the typical and atypical development of numerical cognition (representation of numbers, number concept, basic arithmetic operations), which serves as the starting point for the design and research in the testing process of *MiniMath* as well as the framework of interpretation of empirical results.

The evolution theory of David **Geary** (1995) about the cognitive development differentiates between the species-related cognitive competencies that are mostly determined by biological factors (e.g. language), and the culturally determined cognitive abilities, realized in a specific cultural context (e.g. reading). Infant research indicates, that in the area of mathematics there are competencies which can be regarded as biologically primary abilities, they are innate and universal. The exact scope of these is still debated, but more and more data suggests that the *approximate number system* (ANS) and maybe the *object-tracking system* (OTS), which helps to identify small (1-4) numerosities (*subitizing*) are responsible for it (Piazza, 2010). These two can be regarded the *core knowledge systems of number*, defined as domain-specific representational priors that guide and constrain the cultural acquisition of novel representations (numerals, number-words) (Spelke & Kinzler, 2007).

Geary termed the most school-taught competencies *biologically secondary abilities*, which are culturally determined, although built from more primary systems. Understanding of the base-10 system, ability to solve complex arithmetic problems, and multi-step word problems can be acquired by deduction or by learning from others. The effectiveness of this learning process is determined by general intellectual abilities, and by educational and instructional practice, that's why the degree to which children master arithmetic varies individually and from one country to the next.

Regarding cognitive development, following the *neuroconstructivist* approach of **Karmiloff-Smith** (2006, see the supposition of *representational redescription*), and based on the *triple code model* of **Dehaene** (1992) as far as the area of number pocessing is concerned, **Von Aster and Shalev** (2007) have worked out the *four-step-developmental model of numerical cognition*. Their theory well matches the above cited thoughts of Geary, and they have formulated predictions regarding the atypical development of numeric abilities which can be tested empirically.

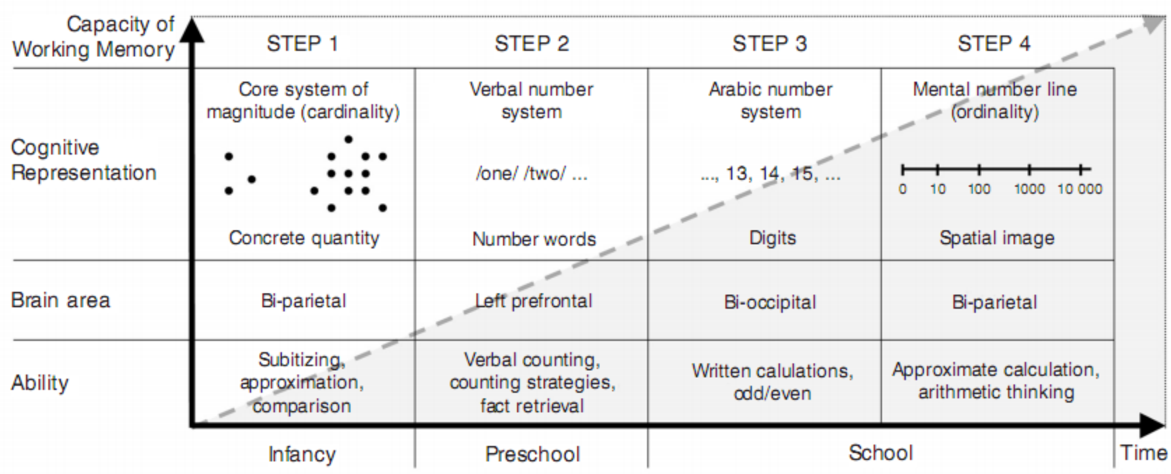


Figure 1: *Four-step-developmental model of numerical cognition. Shaded area below broken line: 'increasing working memory.'*

The central element of the adult number processing system is the analogue representation of quantity, which gives meaning to numbers, the *mental number line*, which presupposes an intact inherited core-system representation of cardinal magnitude (ANS, also called *number sense*), but also requires several domain-general competencies, such as the development of language, attention, and working memory, which makes the construction and automatization of a spatial image of ordinal numbers possible.

The model implies two ways of atypical development of counting abilities which corresponds to the subtypes of DC identified by Von Aster, Schweiter & Weinhold Zulauf (2007) in their longitudinal research:

1. in the case of *pure DC* (1,8%) step 1 fails to be established appropriately, because of genetic vulnerability;

2. in the case of *comorbid DC* (4,2%) children have language and/or ADHD problems, which seem to impede the typical acquisition of number concepts. These concepts range from the linguistic and Arabic number system to the construction of the mental number line (while core system abilities seem to be intact).

This way the authors resolve the opposition between the domain-specific view of DC (see Butterworth (2005) *defective number module hypothesis*) vs. the domain-general theory of *mathematical learning disability* (see Geary and Hoard, 2002), and suggest a sophisticated DC-model, which is in harmony with the complexity of the brain network involved in processing numbers, and which can provide important conclusions for DC diagnostics and therapy (see Chapter 5).

The approach of Robert **Siegler** (Siegler, 1996, 1999), focusing on strategies while studying cognitive development, provided another important theoretical framework. He considered the development of mathematical competencies during pre-school and school age to be continuous (lacking sudden transitions), due to the accumulation of specific knowledge and strategies. Within the *overlapping waves model* children typically know and use a variety of strategies at any one time. With age and experience, the relative frequency of each strategy changes, with some strategies becoming less frequent (cease to be used), while new strategies are discovered and becoming more frequent. Improvements in speed and accuracy that generally accompany learning in the field of arithmetic operations, can reflect at least 4 types of specific strategic changes: introduction of new strategies, increasing use of the most efficient existing strategies, more efficient execution of each strategy, and more adaptive choices among strategies (Lemaire & Siegler, 1995).

Although the research and computer simulation models of Siegler’s research team are aimed at describing typical development of children’s addition, multiplication, estimation skills, their results and conclusions can also be applied to atypical (delayed or disturbed) development of computational abilities. Reasons behind the *maladaptive strategy choice* could not only be cognitive ones, (such as the lack of conceptual understanding) but also socio-cultural factors (e.g. external reinforcement, see Ellis 1997), and in my opinion emotional-motivational factors (e.g. mathematical anxiety, lack of self-confidence) can also be mentioned. That sheds new light on the typical symptom of DC children, that is the application of immature, mechanical backup strategies to the arithmetic operations (see Chapter 5).

4. Research Conducted On the Topic

In the dissertation I describe three research projects, which are connected to the design of *MiniMath* test. The objectives of our preliminary research conducted between 2005-2007 were:

1. to lay down the scientific foundations for the contents of the test,
2. to test whether the tasks of *MiniMath* could methodologically be suitable to indicate turning points of the typical development of number skills, and furthermore, to diagnose mathematics disorder.

The third empirical part presents the results of data acquired during testing the *MiniMath 2.0 computer program* in 2011-12, and sheds light on the ongoing process of further developing the program.

4.1. Age-related differences in number processing reaction time in 3rd and 5th graders

In the first phase of our research we have compared the counting competencies of typically developing children in 3rd and 5th grade by applying the *experimental version of MiniMath,* consisting of eight numerical tasks selected from *MiniMath,* programmed by the *Presentation*® software system. This version did not differ in contents, but differed in appearance and in the way of providing answers: the tasks are not embedded in a framework story, the graphic display is very simple, and in most cases verbal answer is expected, which we have recorded with the help of a voice key. So *experimental version* is closer to the research methods of cognitive psychology, as *MiniMath program.* Data collected with it can directly be compared with results of other experiments, and answering theoretical questions is not made difficult by methodological concerns.

The goal of our study was to investigate whether the stabilization and automatization of basic number skills (which we can expect in this age according to the literature) can be captured on the level of reaction time. We have formulated hypotheses regarding the a**ge-related differences in reaction time** and in **reaction time patterns** (which means the change in reaction time affected by number size or problem-difficulty)

1. We expected significant differences between age groups in counting dots, in multi-digit number reading in timed tests of simple addition, subtraction, and in performance on

computational problems. We expected no difference in 1-digit number naming, and in parity judgement, because these functions have become automatic at an earlier age, which is signified by the lack of *problem size effect* (the reaction time does not increase with increasing magnitude - see Dehaene, Bossini & Giraux, 1993; Butterworth et al. 2001, Verguts et al., 2005).

2. Patterns of reaction time give (some) information about the strategies used to solve arithmetic operations. We have examined the use of *retrieval-strategies* in simple addition and subtraction according to problem-difficulty (see: Siegler, 1988; Geary & Widaman, 1992; Fuson, 1992; Seyler et al., 2003; Barrouillet et al., 2008), the use of the plausibility *strategy* in verification tasks (in additions) (see: Lemaire & Fayol, 1995; Campbell & Fugelsang, 2001), and the use of the *principle of inversion* in inversion problems (see: Stern, 1992; Bryant et al., 1999).

Participants were seventeen 3rd graders (aged 9.3-10.4 yrs) and nineteen 5th graders (aged 11.1-12.3 yrs) from elementary schools in Budapest. They had high-average IQ and working memory score and had no known disturbance of learning or behaviour. Tasks assessing basic numerical skills can be found in *Table 1*.

	Experimental version of MiniMath	**Description of the task**	**Items**
1.	Number naming, number reading	transcoding single- and multidigit Arabic numerals to verbal number words	20
2.	Dot counting	enumerating dots presented simultaneously (1-10) by subitization (1-3), or counting (4-10)	20
3.	Addition-table	answering simple addition problems	12
4.	Verification of addition problems	true/false judgements of correct and incorrect additions (e.g. 14+5=17)	16
5.	Subtraction	answering single- and multidigit subtraction problems	6
6.	Composing and decomposing numbers	composing (4+… =6) and decomposing (5-…=2) numbers up to 15	6
7.	Algorithm inversion	answering inversion problems (A+B-B) and control problems (A+A-B) by the use of inversion principle or multi-step computation	8
8.	Parity judgement	odd/even judgements of single digit arabic numerals (1-10)	10

A *picture naming* task was used to measure general processing speed. This allowed for ruling out the possibility that age-correlated performance improvement was simply due to a general improvement of processing speed.

There was no attempt at statistical analysis of errors for the numerical tasks, due to the very low proportion of errors made (0,39%). Only reaction times for correct answers were calculated and used for statistical analysis. We examined the reaction time patterns in individual tasks with mixed design repeated measures ANOVA, with the between-subject factor of school grade and with the within-subject factor problem-type (according to problem-difficulty), or number size. Results presented in *Table 2*, show the latencies, the supposed solution strategies, and the age-group differences.

RT[1] (ms)	Numerical Task – Problem-type	Strategy used
500	1-digit number naming	retrieval, asemantic route
650	Addition-table – ties (e.g. 2+2)	retrieval
750	Multi-digit number reading	place value, semantic route
800	Dot counting 1-3	subitization
900	Parity judgement	retrieval
1000	Addition-table – simple (<10)	retrieval
1550	Composing/decomposing numbers – simple (<10)	retrieval & transformation (?)
1600	Subtraction – simple, small numbers (e.g. 8-5)	retrieval & transformation (?)
1900	Addition-table – difficult (>10)	retrieval (?)
2400	Subtraction – simple, big numbers (e.g. 57-4)	calculation
2550	Dot counting 4-10	counting (by one?)
2550	Verification of additions (correct trials)	calculation & comparison
2800	Verification of additions (incorrect trials)	calculation & comparison
2850	Composing/decomposing numbers – difficult (>10)	calculation (adding up)
3150	Algorithm inversion	use of inversion principle
3800	Subtraction – difficult (<10)	calculation
4400	Multi-step computation, small numbers (e.g. 5+5-2)	calculation
7500	Multi-step computation, big numbers (e.g. 16+16-5)	calculation, place value

Highlighting indicates age-group differences found

Results show age-specific development in

1) the effectiveness of **counting**,

2) **understanding base-10 number system**, which makes possible the automatization of 3-4-digit number reading, and calculation with multi-digit numbers

3) the effectiveness in **retrieval arithmetic facts** (addition-table, parity),

4) **addition** (<20), as well as in **composing/decomposing** numbers.

No age-related difference has been found in the speed of **subitizing** and of **1-2-digit number naming** (single-digit number naming was fully automatic after reaching third grade age). Nevertheless both groups struggled with **subtraction** and with recognizing the principle of **inversion**, because these required significant semantic elaboration.

According to our cross-section study the tasks of the *MiniMath* can be suitable for a sophisticated measurement of numerical abilities of elementary school pupils. The most important indicator of performance is reaction time, which has to be recorded with a punctuality of milliseconds.

[1] For the sake of clarity the average reaction times of the whole sample have been rounded up in the table.

4.2. Basic Numerical Skills in Children with Severe Developmental Dyscalculia

In the second phase of our research we have tested the numerical skills of children with DC, with the *experimental version of the MiniMath*. We tried to answer the question whether the arithmetic skills deficit of the DC children surfaces, and if it does, in what form in solving the tasks measuring basic arithmetic skills in committing errors and/or in reaction time.

Participants (N=11) of this study were chosen from a child psychiatric referral population (identified by ICD-10 criteria). In order to reduce the number of false positives, we selected children (4^{th}-6^{th} graders), who didn't show any improvement after one year intensive training and we have excluded children with ADHD diagnosis. Our DC sample has been comprised of children with rather severe difficulties: their mathematical age was approx. 7-8 years. The DC sample size is relatively small, but this is not uncommon in this research field (see. Landerl, Bevan & Butterworth, 2004, Rousselle & Noël, 2008, Van Loosbroek, Dirkx et al., 2009, Soltész & Szűcs, 2009, Mazzocco, Feigenson & Halberda, 2011 testing reaction times). The control group was matched on age, gender, and IQ further increases the value of the study.

The characteristics of the sample are shown in *Table 3*.

	DC sample	**Control sample**
Sample size (N)	11	10
Gender	5 boys, 6 girls	5 boys, 5 girls
Age	12;2 yr (±1;2yr)	11;6 yr (±0;4yr)
Grade	4th grade: 2 people 5th grade: 6 people 6th grade: 3 people	5th grade: 10 people
Intelligence	Based MAWGYI-R, corrected IQ 92.27 (±9.72)	Based on SON test, non-verbal IQ 98.9 (±8.24)

Data collection and statistical analysis was carried out in the same way as in our first study. However in the non-numerical *Picture naming task* reaction time of the two groups significantly differed. We can't disentangle the processing stage at which problems (resulting decreased general processing speed) occurs: stimulus evaluation phase, retrieval from semantic memory, or response initiation, but in order to control the difference in baseline, we calculated the fraction $RI_{numerical}/RI_{general}$[2] in three tasks (*Number naming/reading, Addition table* and *Parity judgement).* This ratio indicates specifically the effectiveness of number processing.

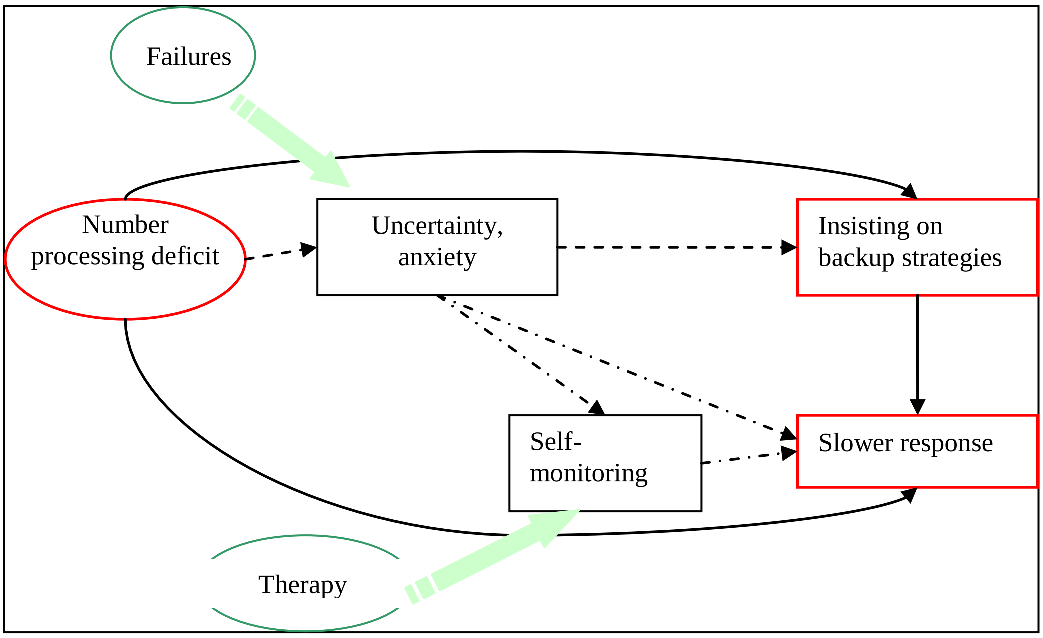
[2] $RI_{numerical}$ is the measured reaction time in the numerical task, $RI_{general}$ is the measured reaction time in *Picture naming* task.

To sum up our results: *persistent DC* children's basic arithmetic skills showed a **specific profile** tested by the e*xperimental version of MiniMath*, which can mostly be described as **developmental delay**. The direct asemantic route was involved for transcoding one- and two-digit numbers, but in case of three- and four-digit numbers, where semantic mediation (syntactic analysis of the numeral) is needed, DC children were slower. During counting and doing basic arithmetic operations DC children increasingly relied on and used mechanically *backup strategies*. The *execution of the algorithm strategy,* especially during subtraction is also slower.

Lower performance of DC children is mostly mirrored in **longer reaction time**. Accuracy (number of correct answers) was deficient only in the most severe cases (two children in our sample), and in the most difficult tasks (difficult subtraction, multi-step computation). We assume, that most DC children have a **more general** (not exclusively number-specific) **deficit**: *verbal working memory*, *executive functions,* (see limited digit span; inhibition problems), and *accessibility of non-numerical and non-verbal semantic information* (see picture naming, and parity judgement) can be sufficient.

Our results confirm the *four-step-developmental model of numerical cognition* (Von Aster & Shalev, 2007). In cases of delayed development of working memory, even with an intact core system of magnitude, the representational redescription fails, which inhibits the establishment of mental number line, the development of number concept and conceptual understanding of arithmetic operations. Based on our research we concluded, that working memory, and (some components of) the execution system can be affected also in DC with no comorbide ADHD).

Regarding the *insistence on backup strategies*, I emphasized the role of mathematical anxiety and of mathematical self-confidence in strategy selection (see *Figure 2.*). These emotional factors (determined by prior failures in mathematics/in school) can have an important role in explaining the symptoms present in arithmetic operations, partly mediated by increased *self-monitoring* (which can be further strengthened by therapy).



Finally some methodological remarks have to be made. Reaction time studies with DC children and diagnostic tools based on reaction time have to use *various non-numerical control tasks* in order to determine the background deficit and to define the direction of therapy. Such tasks can be

- non-numerical working memory tasks: to measure the capacity of verbal working memory, and execution system;
- picture naming or colour naming tasks: to measure the speed of access;
- simple or choice reaction time tasks: to grasp the general speed of processing.

Reaction time in numerical tasks have to be compared with reaction time measured in the control tasks. That can be done according to several formulae (e.g. calculating ratio, difference), but the use of any is strongly recommendable.

4.3. Testing MiniMath 2.0 Computer Program

In the last empirical section I have interpreted the results from testing the *MiniMath 2.0 computer program* for 5-7 year old children. In the current phase we collect feedback on the usability of the program coming from typically and atypically developing (having general learning disability, or language disorder) children. Regarding the contents of the program our objective was to examine the psychometric properties of the test.

1. Do the tasks indeed measure what we expected (validity): do the numerical and non-numerical cognitive skills, which we want to measure, indeed determine the level and speed of performance?
2. Do the tasks measure these skills reliably (e.g. enough items, guessing can be filtered out, reaction time data not too noisy)?

Another important goal was to identify tasks, which could form the *diagnostic version of MiniMath* test in the future. Weaknesses detected in diagnostic versions's subtests should give relevant information about each child to special education professionals for individualizing rehabilitation planning.

Numerical tasks of *MiniMath 2.0 program* tested in our research and the results of testing are summarized in *Table 5*.

	MiniMath 2.0 program	**Description of the task**	**Result of testing**
1.	Dot counting	enumerating dots presented simultaneously (1-10) by subitization (1-3), or by counting (4-10)	marker of core number competence. Indicators: accuracy, RT slope and intercept (within subitizing and counting range)
2.	Counting series of actions (light flashes)	enumerating light flashes presented sequentially (1-10) by counting, or by estimating	marker of counting speed. Indicator: accuracy
3.	Counting series of actions (light flashes) with sequential response (key presses)	producing equivalent numbers of key presses by remembering the number of target set (1-10) and generating the correct response.	can be left out (too difficult)
4.	Error detection in counting	determining if puppet's count was "OK" or "not OK" based on conceptual understanding of counting principles.	marker of conceptual understanding of counting, also sensitive to visual-spatial attention. Indicator: accuracy, more items needed
5.	Number conservation	deciding wheter transformation changes numerosity of a set or some irrelevant, but salient dimension (colour, size, length)	need to be improved (instruction is misleading)
6.	Comparing numerosity of sets	comparing numerosity of sets (pictures of animals, 1-15) by counting, or by subitizing/estimating	marker of core number competence. Indicators: intercept and slope derived from RTs related to discriminability
7.	Informal understanding of fractions	dividing continuous/discrete objects by 2 or 4, and understanding fractions (half and quarter) of one object/a set.	can be left out (too difficult)
8.	Number naming, number reading	transcoding single- and multidigit Arabic numerals to verbal number words	marker of number knowledge and of accessibility of mental number line. Indicators: accuracy, RT slope and intercept
9.	Visual search of target number	finding target number (among distractor-numbers) by transcoding number words to Arabic digits	need to be improved (reading target number is too difficult)

10.	Understanding of quantity words	selecting the correct picture (from two pictures) by understanding the words many, more, few, less, nothing	sensitive to domain-general cognitive abilities: attention, verbal working memory
11.	Number sequence	recognizing number sequence of Arabic digits	marker of number knowledge, sensitive to spatial attention
12.	Numerical Stroop	comparing simultaneously presented Arabic digits based on either their physical or numerical magnitude	marker of accessibility of small/big number's categories. Sensitive to attention/inhibition. Indicators: accuracy, intercept and slope derived from RTs related to discriminability
13.	Addition-table	answering simple addition problems	marker of arithmetic knowledge and strategy-choice. Indicators: accuracy, RT

5. Conclusions Regarding the Diagnostics And Therapy of Dyscalculia

Taking into consideration the profile of strengths and weaknesses of DC children two subtypes have to be differentiated and training should be instituted according to their specific needs.

1. Longer reaction time in basic numerical tasks is not necessarily a marker of core number processing deficit, or a diagnostic sign for defective number module. In most cases (called *comorbide DC*) this is a result of the disfunction of domain-general sub-systems of the number processing network. Indicator of this DC subtype is intact processing of magnitude in tasks with non-symbolic stimuli, which doesn't require counting.

2. Goal of therapy can be fostering representational redescription, constructing and establishing a mental number line. Besides that teaching supporting strategies is important too, which reduce the burden on verbal processing, attention and memory capacities, while solving more complex mathematical tasks.

3. Diagnostic symptoms of *pure DC* are signs of serious and persisting delay in mathematical development, as well as atypical mistakes, inadequate ways of number processing and of arithmetical problem solving.

4. Therapy for these children could mean to teach them compensation strategies based on their strengths. Unfortunately this method has only moderate effect for children with lower intelligence. To prevent secondary symptoms, and strengthening self-confidence (by focusing on other areas) could be the most important help they receive.

Determinants of maladaptive strategy choice have to be examined and considered as follows:

1. The use of immature strategies does not always have a *diagnostic value*. Atypically developing children in general (also children with language/reading disorder or with lower intelligence), and children with high anxiety, or with strong motive to avoid failure also prefer to use backup strategies.

2. It would be necessary to minimize the deterioration of performance due to mathematical anxiety, mediated by maladaptive strategy choice (e.g. avoiding retrieval strategy, or counting when subitizing/estimating would also produce correct result) during the examination of mathematic skills. This goal is served by establishing a friendly diagnostic setting and providing tasks which instead of recalling the school environment evoke a computer game (as we have tried in designing *MiniMath*). Direct observation and recording of test behaviour is useful to determine the indicators of applied strategies.

3. Instead of training the mechanical use of backup strategies, fostering the adaptive strategy choice has to be undertaken. Improving meta-cognitive skills, analysing constraints and benefits of various problem-solving methods, their explicit formulation, reinforcing the use of more mature strategies (even despite incorrect result) would be crucially important for low achieving students.

I have argued in my thesis that results of developmental cognitive neuroscience can get us closer to understand the typical and atypical development of mathematical skills, and to describe the different variations of dyscalculia. This view provides new aspects on both diagnostics and therapy, but there’s still a long way to go until it yields methods and instruments that can be used in practice. I hope that designing *MiniMath* is an important step towards that goal.

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